Converting NFAs to DFAs

Lecture 7
Section 2.3

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Outline

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The Algorithm

- We will convert a DFA $M = \{Q, \Sigma, \delta, q_0, F\}$ to an NFA $M' = \{Q', \Sigma', \delta', q_0', F'\}$, where
  - $Q' = \mathcal{P}(Q)$
  - $\Sigma' = \Sigma$
  - $F' = \{S \subseteq Q \mid S \cap F \neq \varnothing\}$
- The start state q_0' and the function $\delta' : \mathcal{P}(Q) \to \mathcal{P}(Q)$ will be described next.

The Algorithm

Definition (λ-Closure)

The λ-closure of a state q, denoted $\mathrm{Cl}(q)$, is the set of all states reachable from q by using only λ-moves. The state q itself is automatically included in $\mathrm{Cl}(q)$.

Definition (λ-Closure of a Set)

The λ-closure of a set S to be $\mathrm{Cl}(S) = \bigcup_{x \in S} \mathrm{Cl}(x)$.

The Algorithm

- The start state of M' is $q_0' = \text{Cl}(q_0)$.
- For any state $S \in \mathcal{P}(Q)$ and for any $a \in \Sigma$, define

$$\delta'(S) = \bigcup_{q \in S} \text{Cl}(\delta(q, a)).$$

Outline

1. Building a DFA from an NFA

2. Examples

3. Assignment

Example

Example (Even Number of **a**'s and **b**'s)

- Let $\Sigma = \{\mathbf{a}, \mathbf{b}\}$.
- Let $L_1 = \{w \in \Sigma^* \mid w \text{ contains an even number of } \mathbf{a}\text{'s}\}$.
- Let $L_2 = \{w \in \Sigma^* \mid w \text{ contains an even number of } \mathbf{b}\text{'s}\}$.
- Convert the NFA that accepts $L_1 \cup L_2$ to a DFA.

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- Convert the NFA that accepts $L_1 L_2$ to a DFA.

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- In the last example, process **ababb**, **abaabb**, and **aababb**.
- Describe the language $\overline{L_1}$.

Outline

Assignment

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- Section 2.2 Exercises 19, 23
- Section 2.3 Exercises 3, 4, 5, 7, 9, 10, 16.